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MATHEMATICAL MODELING OF MAGNETIZATION IN BISTABLE AMORPHOUS MICROWIRES

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ABSTRACT

A mathematical model that describes the process of the reversal magnetization of an amorphous microwire with the help of a large Barkhausen jump is proposed. The model has been estimated with regard to the optimization of the signal-to-noise ratio.

Keywords: **domain wall, spline function, stochastic resonance, residual stresses.**

Introduction

A property of a cast glass-coated amorphous micro- and nanowire (CGCMNW) with the positive magnetostriction and a wide range of microwire strand radii (from 50 to 0.1 μm) to be reversely magnetized in a single large Barkhausen jump (LBJ) is sometimes used to indicate the magnetic field (see more in [1]). The aim of this work is to survey the available data concerning LBJ (for instance, see [2–8]); emphasizing the results which are fundamentally new. This will help to outline the directions for further development, namely, of the method of the phenomenological equation of motion of a domain wall (DW) [7, 8]. As it is difficult to choose unequivocally a model of the phenomenological motion equation for a DW, one needs to use some notions of micromagnetism in order to simulate probable physical processes [5, 6, 9]. We intend to find a consistency between the proposed mathematical model and parameters of a DW [5, 6] for physical interpretation of the processes that describe LBJ. The outline of the paper is as follows. First, we will present the previous theory [2–4], modified in [6–8]. Then we are briefly describing the theory of the CGCMNW DW [5], adding some new important results. Next, we substantiate a modification of the phenomenological approach to dynamics of a DW in order to produce a qualitatively new phenomenon – stochastic resonance that has been already offered in [8] for CGCMNW for a particular case. In order not to overload the text, we present the detailed calculation data for residual stresses, partially described in [5], in Appendix 1. The exchange energy formula is calculated in Appendix 2.

Phenomenological Mathematical Model of the Motion of a Domain Wall

Dynamics of a DW is studied using the solution of the Döring equation (see, for instance, [2–4]):

$$m_{ef}\ddot{x} + \beta\dot{x} + F_i(x) = 2M_S SH(t), \quad (1)$$

where m_{ef} is the effective mass of DW, β is the phenomenological attenuation coefficient, $F(x)$ is the force function that characterizes the action of a magnetic matter on DW. This force describes the gradient of the potential relief (GPR).

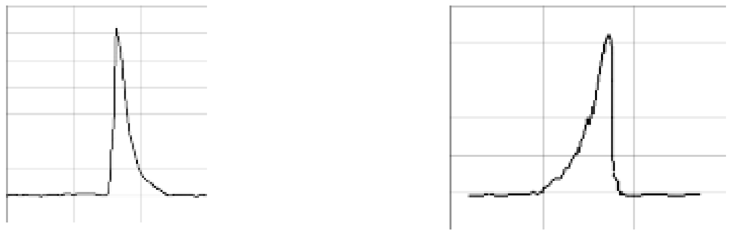


Fig. 1. Relaxation (at the left) and acceleration (at the right) forms of the *EMF* pulses at LBJ that are fixed on the CGCMNW samples with the iron-based strand material (with positive magnetostriction) [2–7]. We are mainly interested in the relaxation form of the DW motion.

According to [5, 6] it is assumed that $F(x)$ appears primarily due to residual stresses. An external field with the intensity H exerts the $2M_S SH$ pressure on the 180° DW (M_S is the saturation magnetization; S is the area of a DW). The generalized coordinate x is the analog of the radial coordinate of a cylinder but its range of definition may be formally extended from $-\infty$ to ∞ that is determined by the correlation of the calculation data with experimental results.

According to [2–7] the *F(x)* force that affects DW can be described in mathematical formalism of spline functions (SF). These functions can be presented in the simplest form as, for instance, in [7]:

$$F_1(x) = \begin{cases} 1)\ \dfrac{F_0}{x_0^2}x^2 \text{ при } x < 0 \\ 2) -\dfrac{2F_m}{c_i^3}x^3 + \dfrac{3F_m}{c_i^2}x^2 \text{ при } 0 \le x \le c_i; \\ 3) -F_m\dfrac{(x-c_i)^2}{(R_m - c_i)^2} + F_m \text{ при } x > c_i, \end{cases} \quad (2)$$

where R_m corresponds to the radius of the microwire strand, and c_i is the dimension of the range, in our case it is similar to the dimension of the plastic strain range (see below).

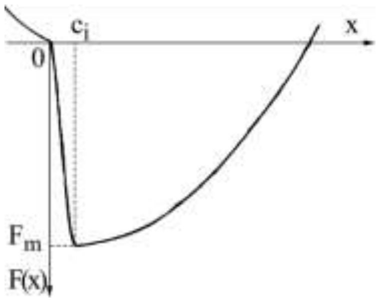


Fig. 2. Form of *F(x)* function (2) which approximates SF for the values of F_m and c_i parameters in the case of the relaxation form of the *EMF* pulse for LBJ.

We assume that the DW moves from the CGCMNNW center to the outer surface of the strand. As the *EMF* pulse waveform (see Fig. 1) in the measuring coil should vary with the rate of the DW motion, this *EMF* waveform should correspond to the *F(x)* function (Fig. 2).

In this case the quantities F_m, c_i and others, which determine the coefficients in polynoms, can be easily calculated from the LBJ oscillograph records (see [2–4] and the text below. Thus, the computations show that the model qualitatively describes both the relaxation and acceleration mechanisms (for more detail see below and in [3, 4]), i. e. the pulses with a steep rise edge and a flat droop or with a flat rise and a steep droop (Fig. 1).

It was found that the values of coefficients in SF should also be in agreement with micromagnetic parameters; this will be considered below. We will show that it is not sufficient to present SF in the form of formula (2) in order to simulate LBJ in CGCMNW. Some modified theory is reported. But at first we are going to discuss some earlier works and demonstrate a good agreement between a simpler variation of the theory with experiment, in particular, with vicalloy actuations [3, 4].

Comparison of Calculations Using Phenomenological Model with Experimental Results

In earlier works [3, 4], when describing shift mechanisms for ferromagnetics mathematically, researchers used a simplified SF, i.e. F(x) that represents GPR. This function had the following form:

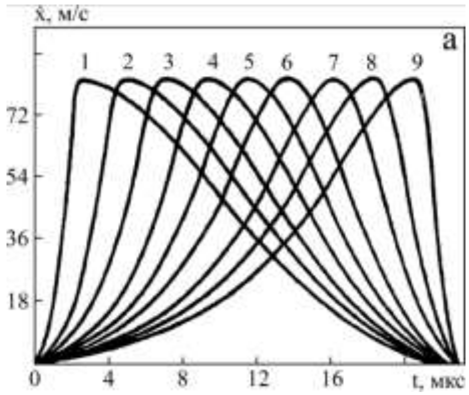
$$F_2(x)=\begin{cases}-\dfrac{2F_m}{c^3}x^3+\dfrac{3F_m}{c^2}x^2\\ \text{at }\ x\le c,\\ -\dfrac{F_m}{(R_m-c)^2}x^2+\dfrac{2F_mc}{(R_m-c)^2}x+\dfrac{F_mR_m(R_m-2c)}{(R_m-c)^2}\\ \text{at }\ x>c.\end{cases},\qquad (2a)$$

This approximation of SF made it possible to vary physical parameters which changed the character of the DW motion and the *EMF* signal. For this purpose they changed the depth F_m, the radius R_m and the position of the minimum *c*. Thus (see Fig. 3a, b):

a) fitting the parameters of SF it is possible to obtain the similarity of the calculation relationships for the DW motion rate which is proportional to *EMF*, i.e. to the quantity *e(t)* that is estimated experimentally using a measuring coil (See Fig. 3a);

b) the simulation data received for magnetic (vicalloy) wires are in quite good agreement with the *EMF* impulses from LBJ (see Fig. 3b);

c) in magnetic amorphous microwires and particularly in CGCMNW this method allowed an adequate description of the acceleration form of the DW motion. However, in order to describe the relaxation form of the pulse with a sharp initial edge it is necessary to take into account the starting motion velocity of DW, which can be found from the form of SF as presented in formula (2)



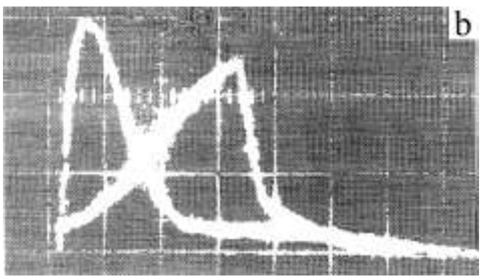


Fig. 3. Visual aid of the forms of the velocity functions *x'(t)* (a).

Experimental oscillograms of *EMF* *e(t)* estimated using a measuring coil [3, 4] (b).
Depending on the SF parameters one observes the relaxation (first) or acceleration (ninth] forms of pulse of *EMF* from *LBJ*, which are also recorded on the CGCMNW samples with the iron-based strand material (with positive magnetostriction) [6].

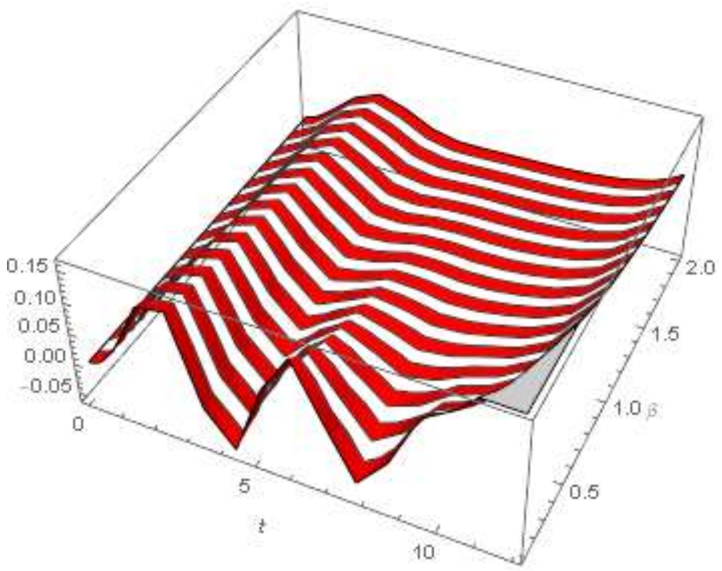


Fig. 3a. Example of calculation of forms of movement DW under formulas (2a)

Comparison of Micromagnetic Parameters of a Domain Wall with Mathematic Model

As noted above, the calculation of residual stresses in CGCMNW (see [5] and appendix 1) as well as the simulation of the motion of DW in GPR *F(x)* are carried out for a model in which the cross section of CGCMNW is conventionally divided into some segments (Fig. 4). Let us perform a micromagnetic foundation of the proposed approach to the dynamics of DW.

Let us write the dimensions and energies of DW estimated for every segment (the residual stresses in these segments are calculated in appendix 1).

According to the model [5, 6] the process of the DW motion begins from the central region inside the cylinder within area 3 (Fig. 4). There appears a reversal magnetic center.

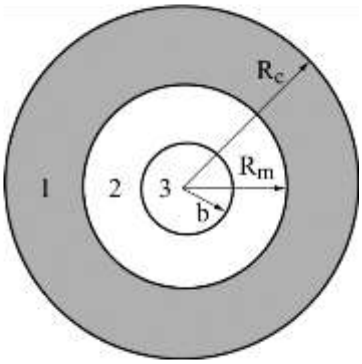


Fig. 4. Cross section of the CGCMNW microwire: R_c is the external radius of the glass envelope, r_m is the radius of the metal strand, b is the radius of the plastic deformation boundary.

This very center is an initial region described in (2); its micromagnetic characteristics are calculated as follows:

(1) In the model of CGCMNW [5, 6] it is energetically more useful for DW to originate in the region with r<b (b ~ 10^{-6} m is the radius of plastic deformations (see [5] and Fig. 4)) where the anisotropy energy is smaller. Let us estimate the order of magnitude of the DW dimensions Δ_i, that for example determine the dimensions c_i. The formula for the exchange interaction energy has a standard form [5, 9], which is the same for all segments (see appendix 2):

$$W_A \approx \frac{A}{\Delta_i}, \quad (3)$$

where $A \sim 10^{-11}$ J/m is the exchange energy constant, $i = 1...3$ is the index that characterizes the position of DW in the scheme of the CGCMNW cross section. In the case of the known model in the Landau–Lifshitz theory (LL) the anisotropy energy can be presented in the following form:

$$W_{a(LL)} \approx \lambda\sigma\Delta_{LL}, \quad (3a)$$

where $\lambda \sim 10^{-6}$ is the magnetostriction, and σ is for the residual stresses (here the average value of them). Minimization of these competing energies is known to give a classic result of the LL theory for the dimensions of the DW and its energy [5, 9]:

$$\begin{aligned} \Delta_{LL} &\approx (A/K)^{1/2} \approx \left(10^{-5} \div 10^{-6}\right)\text{m} \\ W_{LL} &\approx (AK)^{1/2} \approx \left(10^{-4} \div 10^{-5}\right)\text{J/m}^2. \end{aligned} \quad (3b)$$

If, for the anisotropy energy, one uses the dependence on Δ_3 (the subscript 3 means the position of DW) in the center of the CGCMNW metal strand, which is prescribed by the real form of the residual stresses (see appendix 1 and [5, 6]):

$$W_{a3} \approx \lambda K b \times \ln\left(\frac{\Delta_3}{b}\right), \quad (4)$$

where K ~ 10^8 Pa is the constant of the plastic deformation stresses, then the minimization of the total leads to a simple expression received previously (see [5, 6]:

$$\Delta_3 \approx \frac{A}{\lambda K b} \leq 10^{-7} \text{ m}. \quad (4a)$$

The value of the DW dimensions is an order of magnitude less than the characteristic diameter of CGCMNW. This is important for adjustment of the model as the dimensions of the DW could not be more than the diameter of the microwire strand (being the case in LL theory). We obtain the value of the energy density of the considered DW as follows:

$$W_3(\Delta_3) \approx \frac{A}{b} \leq 10^{-5}\ J/m^2\ . \quad (5)$$

According to formula (5) the field of the start does not depend on anisotropy but is governed by the process parameter b which determines the order of magnitude of the polynomial coefficients in SF. By formulas (4) and (5) it is also possible to estimate the dimension of the nucleation center of a cylinder domain wall that is generated I other amorphous materials as well. It follows that the energy estimations of domain nucleation can differ markedly from the estimations in the LL theory towards smaller energies as necessary the magnetic reversal centers to generate; this was observed experimentally. If we go back to formula (2), the initial segment is defined by the first function; this corresponds to a harmonic oscillation or damped motion with no oscillation. In fact, it is the damped form of motion in the case of DW [2--4, 9]. Thus, the motion point is not very important within the proposed model.

Note that formulas (3) – (5) are applicable to any magnetic materials in which magnetic nucleation centers appear at the particular types of dislocations. More on the energy of anisotropy associated with dislocations is in [9] (p. 48). This anisotropy energy:

$$W_d \approx \lambda K_d b_d \times \ln\left(\frac{\Delta_d}{b_d}\right), \quad (5a)$$

where b_d is the dislocation size, $K_d \sim K$.

2) Let us consider the case which is possible due to the peculiarity of the dependence of residual stresses on the radial coordinate of the wire. This very case describes the motion of the DW in the center of the microwire (in the middle of segment 2 in Fig. 4). Let us prescribe the anisotropy energy in the following form (according to [5] and appendix 1):

$$W_{a2g} \approx K_{2g}\left(\frac{\Delta_g}{b}\right)^n, \quad (6)$$

where

$$K_{2g} \approx \lambda P b^2,$$

and the quantity n is defined below. For the value of DW and the DW specific energy we obtain:

$$\Delta_g \approx b\left(\frac{A}{nK_{2g}}\right)^{1/(n+1)}$$
$$W_{a2g}\left(\Delta_g\right) \approx K_{2g}\left(\frac{A}{nK_{2g}}\right)^{n/(n+1)}. \quad (7)$$

Formulas (7) for a particular case ($n = 1$) is the same as for the LL theory when the domain wall and the domain wall energy is known to be determined by the geometric mean of two characteristic dimensions associated with the exchange interaction and anisotropy. On physical grounds $n > 0$ for this formula, otherwise there is no DW.

Please note that it is the simulation of the relaxation pulse in the previous and this region which is the bottle neck for the function *F(x)*. At that, the acceleration mechanism of the DW motion (Fig. 1) is simulated better than that of the relaxation mechanism even if we assume that the initial rate of the DW is zero. Thus, this is the reason to suggest that there is some accelerated motion of DW in the case of the relaxation process (the first segment of motion in formula (2)).

3) Let us consider the case when DW achieves the region close to the silicate glass junction, i. e. the reversal process is over (the end of segment 2 at the boundary with segment 1 in Fig. 4). Then, the anisotropy energy is constant and is defined by the quantity~ λP determined in [5] and appendix 1. Thus, we assume that this very quantity~ λP determines the constant member F_m in (2).

The quadratic term in (2) takes into account the fact that the dimensions of the DW in motion are increased due to its cylinder shape. The following form of the anisotropy energy is considered:

$$W_{a21} \approx \lambda P\left(\frac{\Delta_{21}^2}{R_m}\right), \quad (8)$$

where R_m (the radius of the CGCMNW strand) is introduced to keep the balance of dimensions of W_A and W_{a21}. Then the size of the DW and its energy will be defined as follows:

$$\Delta_{21} = \left(\frac{AR_m}{\lambda P}\right)^{1/3} \approx \left(10^{-6} \div 10^{-8}\right)m,$$
$$W_{21}\left(\Delta_{21}\right) = \left(\frac{A^2\lambda P}{R_m}\right)^{1/3} \approx 10^{-4}\ J/m^2. \quad (9)$$

The start field H_{c21} (corresponding to W_{21}) is larger than the start field in the third region H_{c3} (corresponding to $W_3(\Delta_3)$). The dependence of H_{c21} on the radius of the microwire strand R_m and that of the glass envelope R_c, using for parameter P formulas (15) from appendix 1, has the following form:

$$H_{c21} \approx \sqrt[3]{\frac{\lambda k x}{R_m\left[\left(\frac{k}{3}+1\right)x+\frac{4}{3}\right]}},$$
$$k \approx 0,3-0,5, \tag{10}$$
$$x = \left(\frac{R_c}{R_m}\right)^2 - 1.$$

4).Nowadays, when studying the reversal magnetization of CGCMNW it is often supposed that one can use a model of the DW motion which is called the “head-to-head” model [10].

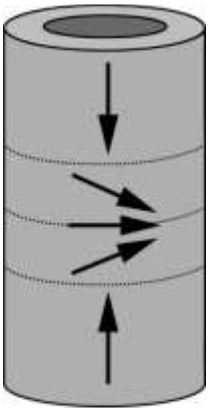


Fig. 5. Example of magnetic structure which corresponds to the “head-to-head” type.

For instance, a simplified version of such model can be considered as an extension of a spherical DW which is in a cylindrical amorphous magnetic matrix within the region where anisotropy differs from the anisotropy of the matrix.

If, “same as previous”, the energy, corresponding to the anisotropy of DW, will change only due to the increase in the DW surface, we will write thus energy as a function of the DW width in the following form:

$$W_{as} \approx K_s\left(\frac{\Delta_s^3}{R_s^2}\right), \tag{11}$$

where, “same as previous”, the multiplier R_s^2 (R_s is the characteristic radius of the action of anisotropy close to the dimension of the domain wall) to keep the balance of the dimensions of the quantities W_A and W. Minimization of functional leads to the following formulas for the dimension of DW and its energy:

$$\Delta_s = \left(\frac{AR_s^2}{K_s}\right)^{1/4} \approx \left(\Delta_{LL} R_s\right)^{1/2},$$
$$W_s\left(\Delta_s\right) \approx \frac{\left(W_{LL}A\right)^{1/2}}{\left(R_s\right)^{1/2}}. \tag{12}$$

The DW dimension is here similar to the dimension of the domain in the LL theory with a characteristic “sample length”, in this case the quantity R_s plays its role, i. e. the region where the anisotropy of the spherical inclusion considerably differs from the anisotropy of the matrix. The energy of DW is proportional to the geometric mean of the DW energy in the LL model and the exchange energy, but at the same time it is inversely proportional to (R_s) $^{1/2}$. You might as well speak about the "rigidity" of these domains which are not easily suppressed by a field opposite to the anisotropy field of the spherical inclusion. The existence of such domains in CGCMNW, the reversal magnetization of which can considerably change the loop rectangularity, has not been observed experimentally.

Please note that results 2) and 4) are presented here for the first time and have never been discussed so far.

Modified Mathematical Model of the Motion of the Domain Wall

Numerical calculations using function (2) and the parameters of the CGCMNW DW are not in conformity with a number of experimental results and the presented theoretical micromagnetic model. It is suggested to create another phenomenological model of the motion of DW when some considerable differences of CGCMNW from the used specially deformed samples of vicalloy alloy wires will be taken into account [2--4]. It was shown in [7] and confirmed in the present work that approximation of *F(x)* in the form of (2) and (2a) is not sufficient for an adequate simulation. Another approximation of GPR for SF is proposed in [8] which seems to be most adequate and convenient in order to treat physical phenomena in CGCMNW. SF makes up two potential wells separated by an energy barrier. Note that in [8] there only a particular case of symmetric wells hardly realizable in CGCMNW is considered. Let us present SF in a general form:

$$F_3(x)=\begin{cases}1)-F_{m1}\dfrac{(x-c_k)^2}{(R-c_k)^2}+F_{m1}\ at\ x<-c_k;\\ 2)-\dfrac{2F_{m1}}{c_k^3}x^3+\dfrac{3F_{m1}}{c_k^2}x^2\ at\ -c_k\le x<0;\\ 3)-\dfrac{2F_{m2}}{c_i^3}x^3+\dfrac{3F_{m2}}{c_i^2}x^2\ at\ 0\le x\le c_i;\\ 4)-F_{m2}\dfrac{(x-c_i)^2}{(R-c_i)^2}+F_{m2}\ at\ x>c_i.\end{cases} \tag{13}$$

Using a gradient SF function with two minimums it is possible to examine this bistable system for the case of the existence of a stochastic resonance with quasi-static reversal magnetization of CGCMNW. As strongly damped motions are studied when the DW mass is small enough in comparison with the effect on DW of the friction force, instead of dynamic equation (1) let us consider as first approximation an equation in which acceleration is omitted for simplicity:

$$\beta\dot{x}+F_3(x)=2M_S SH(t)+A\sin(\omega t)+g(t), \tag{14}$$

where $A\sin(\omega t)$ is the monochromatic force that initiates the transition from one state to another, *g(t)* is the stochastic force that appears due to the magnetic noise field. This allows simpler integration of the equation motion of DW. The amplitudes of the determinate forces are suggested to be small enough. A barrier jump between two minima is, in particular, performed owing to the stochastic force. We need to find the process rate that determines the amplitude of the signal. It is also important to estimate the signal/noise relation.

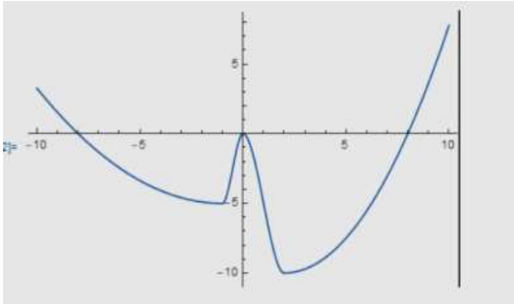


Fig. 6. Typical form of function *F(x)* .

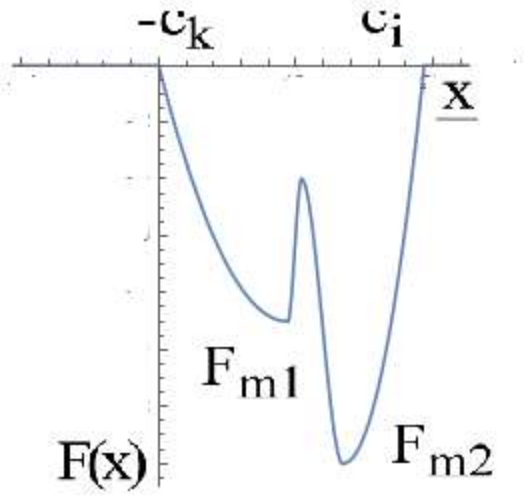


Fig. 6a. Form of function *F(x)*, which approximates SF, depending on the values of its parameters F_{m1}, F_{m2} and c_i, c_k. (In [8] this function had a symmetric form).

The preliminary results of numerical calculations present a qualitative picture of the outlet increase in the signal/noise relation at certain correlations of the monochromatic force frequency and the gradient function parameters. The range of these frequencies is in the region of 10 KHz that is of some interest in order to improve the operation of miniature sensors of a magnetic field from CGCMNW. A more detailed comparison with the experiment will be made in some other paper.

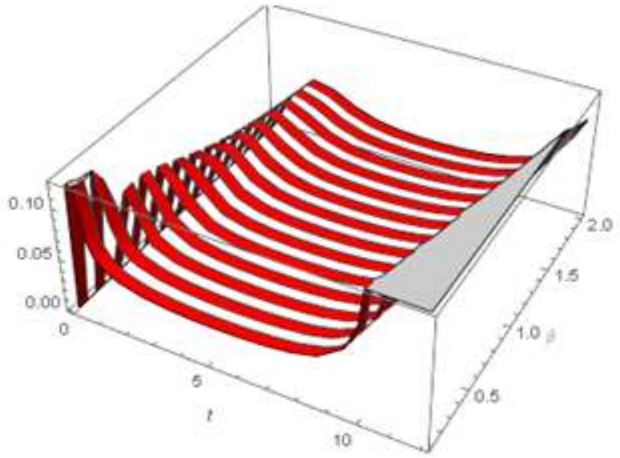


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Fig. 6b.. Example of calculation of forms of movement DW under formulas (13)

Conclusions

1) We have analyzed the possibility of using the Döring equation applying the fields of forces described by GPR with the help of SF proposed in previous works [2–7]. Those SFs, depending on the value of their parameters, can describe the relaxation and acceleration forms of the DW motion. However, to present a number of effects and to compare them with micromagnetic calculations the proposed form of SF should be modified; this has been performed in the closing paragraph of this paper.

2) The model of the phenomenological equation of the DW motion has been compared with the micromagnetic calculations. Thus, the can be substantiated the micromagnetic structure of CGCMNW and the form of SFs which are used to study the DW dynamics.

3) The proposed version of GPR for SF makes it possible to study if the stochastic resonance can exist for this system. An opportunity to observe experimentally the stochastic resonance for the phenomenon of the reversal magnetization of CGCMNW is of great theoretical and practical importance. At present, there exist a rather limited number of systems in which stochastic resonance is present.

4) The proposed theory differs radically from the existing theories of DW dynamics [11–13] which are applicable to amorphous ribbons or films but do not take into account the specific nature of CGCMNW. In our opinion, this fact determines the scientific importance of this work, which includes the previous results and new presentations. Note, that for amorphous wires produced by other technologies (see, for example. [14]) other analytic models of the DW dynamics are necessary.

Appendix 1

Let us formulate equations for residual stresses in CGCMNW on the base of a simple model for cooling of a cylinder and a cylindrical surface with different thermal expansion coefficients but no thermoplastic relaxation during cooling. This model (which may be called the Poritzky--Hull--Burger model [15]) is widely used to calculate stresses which appear in macroscopic glass-to-metal seals.

For radial ($\sigma_{r(0)}$), , tangential ($\sigma_{\varphi(0)}$) and axial ($\sigma_{z(0)}$) stress components there have been previously obtained the following formulas (see, for instance, [5]):

$$\sigma_{r(0)} = \sigma_{\varphi(0)} = P = \sigma_m \frac{kx}{\left(\frac{k}{3}+1\right)x+\frac{4}{3}},$$
$$\sigma_{z(0)} = P\frac{(k+1)x+2}{kx+1}, \qquad (15)$$
$$x = \left(\frac{R_c}{R_m}\right)^2 - 1,$$

где E_i – модули Юнга (металла (i = 1) и стекла (i = 2)).

where $\sigma_m = \varepsilon E_1$, $\varepsilon = (\alpha_1 - \alpha_2)(T^* - T) \approx 5\cdot10^{-3}$, α_i is the thermal expansion coefficients (TEC) of metal (i = 1) and glass (i = 2); T^* is the composite chilling point within the metal--glass contact area ($T^* \sim 10^3$ K); T is the temperature of the experiment; R_m is the radius of the microwire metal strand; R_c is the outer radius of the microwire glass envelope;

$$k = \frac{E_2}{E_1} \approx 0,4 \div 0,5,$$

where E_i are the Young's moduli of metal (i = 1) and glass (i = 2).

The longitudinal stress is the highest, thus the longitudinal magnetic structure is confirmed, that is:

$$\sigma_{z(0)} \approx (2\text{–}3)P,\ \sigma_{z(0)} > \sigma_{r,\varphi(0)},$$

and the maximum of P is determined as the following:

$$P \to 0{,}5\sigma_m \approx 10^9 \text{ Pa}.$$

As the analyzed model suggests a simultaneous shilling of the metal rod and glass envelope, it is reasonable to use formulas (15) as an initial approximation (naturally, with allowance for all the drawbacks of this approach). The voltages, calculated in (15), will be considered to work on the surface of the CGCMNW metal strand (at any rate, in the order of magnitude).

More complicated and more adequate model of calculation of residual stresses on the surface of a metal strand was considered in [16]. There the influence of an oxide layer was taken into account that appears between the internal surface of the silicate glass and the external surface of the metal strand (see the dot-and-dash line in Fig. 7). A general solution of this problem can be only numerical. However, the correction to the estimations of the residual stresses (see for further detail [16]) leave the main qualitative results which follow from (15) unchanged.

It should be pointed out that in order to calculate a domain structure for CGCMNW one should take into account the relaxation processes in the microwire strand which lead to the dependences in σ_r and σ_φ of the r - coordinate of the cylinder. Let us initially consider a simple variant of this theory of calculation of stresses using the theory of elasticity, first disregarding the plastic relaxation range and then with regard to the plastic relaxation [5]. For a long cylinder, it is sufficient to introduce the radial deformation which obeys the following equation (the Lamè problem, see [5, 17--19]):

$$u'' + \frac{1}{r}u' - \frac{1}{r^2}u = 0. \qquad (16)$$

Equation (16) enables us to solve a problem on deformation of an infinite cylindrical figures with the internal radius $\vec{r}_1$ and the external radius $\vec{r}_2$. Let pressures P_1 and P_2 operate on the cylindrical surface of this figure. The solution of this problem for residual stresses in each envelope can be presented in the following form [5, 17--19]:

$$\sigma_r = P_1' - \frac{c_1}{r^2}, \qquad \sigma_\varphi = P_1' + \frac{c_1}{r^2}, \qquad (17)$$

where quantities P_1' and c_1 are determined from the boundary conditions for stresses. In our case:

$$P_1' = \frac{P_2 r_2^2 - P_1 r_1^2}{r_2^2 - r_1^2}, \qquad c_1 = \frac{P_2 - P_1}{r_2^2 - r_1^2} r_2^2 r_1^2. \qquad (18)$$

It is assumed that the stresses (that is the pressures which are applied to the cylindrical surface) are (in the case of CGCMNW) tensile being of importance in choosing a sign. In (17) and (18) the presented quantities are positive, if

$$P_2 \ge P_1.$$

This is the case in CGCMNW. It follows thence an important relation:

$$\sigma_r \le \sigma_\varphi. \qquad (19)$$

Presenting a microwire strand in the form of coaxial cylindrical surfaces and prescribing external boundary conditions, one can construct recurrent relations which allow numerical calculation of the residual stresses. The conditions of peak stresses on the strand surface which can be estimated from (15), are assumed the boundary conditions. It follows from the numerical calculations that the residual stresses for CGCMNW in next layers decrease (in absolute magnitude) remaining tensile. Analytical dependences of the residual stresses on the cylinder radius coordinate are important for calculation of the DW width. They are presented (according to [17--19]) in the following form:

$$\sigma_{r(1)} \approx P\left[1-\left(\frac{b}{r}\right)^2\right], \quad \sigma_{\varphi(1)} \approx P\left[1+\left(\frac{b}{r}\right)^2\right], \qquad (20)$$

where b is the minimum limiting value of the radius $\vec{r}_1$ inside the metal strand when the plastic relaxation is negligible. The parameter b is a phenomenological parameter that should be estimated from experimental data. Unlike (15), residual stresses (17) and (20) take account of the fact that some relaxation processes take place in the center of the microwire. Assuming that the range with the boundaries from $\vec{r}_1 \equiv b$ to the external radius $\vec{r}_2 \equiv \vec{R}_m$ is the region of elastic stresses, then it is simple to obtain from (17)--(20) the following equilibrium equation [17--19]:

$$r\left(\frac{d\sigma_r}{dr}\right) = \sigma_\varphi - \sigma_r. \qquad (21)$$

What is more, there is fulfilled the relation, following from Hooke's law, that the sum of radial and tangential stresses at the prescribed radius of the cylindrical surface is constant:

$$\sigma_r + \sigma_\varphi = 2P. \qquad (22)$$

Let us evaluate the stress $\sigma_{z(1)}$ from the known components of $\sigma_{r,\varphi\,(1)}$:

$$\sigma_{z(1)} \approx \nu 2P. \qquad (23)$$

For zero problems (the case of formula (15)) it is fulfilled that:

$$\sigma_{r(0)} = \sigma_{\varphi(0)} = P. \qquad (24)$$

In order to bind the solution to (15) more accurately, one can add the constant stresses $\sigma^0_{r,\varphi,z(1)}$ to the general solution.

Then (17) and (20) can be rewritten in the following form:

$$\sigma_{r(1)} \approx P\left(1-\frac{b^2}{r^2}\right)+\sigma^0_{r(1)},$$
$$\sigma_{\varphi(1)} \approx P\left(1+\frac{b^2}{r^2}\right)+\sigma^0_{\varphi(1)}, \qquad (25)$$
$$\sigma_{z(1)} \approx \nu\left(\sigma_{r(1)}+\sigma_{\varphi(1)}\right)+\sigma^0_{z(1)},$$
$$\sigma^0_{r(1)} = \sigma^0_{\varphi(1)} \approx 0,5\left(\sigma^0_{z(1)}\right),$$

if $\nu \to 0,5$ (and $b \le R_m/2$).

Please note that the analytic extrapolation of numerical calculation has the form:

$$\sigma_{r(12)} \approx A_{12} - B_{12}\left(\frac{r}{b}\right)^n, \qquad (26)$$

where $-1 < n < 2$.

Elastic relaxations within the $\vec{r} < b$ zone can be taken into account, using the Airy function $\Phi(r)$ (see in more detail in [5, 17--19]), which in our case allows the calculation of the residual stresses with regard for plastic relaxation. We restrict ourselves to a centrally symmetrical case when the Airy function depends only on the radial coordinate. In this case the formula $\Phi(r)$ of the bind with the residual stresses considerably simplifies [5, 17--19] to:

$$\sigma_r = \left(\frac{1}{r}\right)\Phi'_r, \qquad (27)$$
$$\sigma_\varphi = \Phi''_r.$$

For convenience in integration, a standard conversion of transfer to the variable t (see [5, 17--19]) is performed according to the following formula:

$$t = \ln\{r\}, \qquad (28)$$

and after conversion the equation for the function $\Phi(t)$ has a simple form [5]:

$$\Phi''''_t - 4\Phi'''_t + 4\Phi''_t = 0, \qquad (29)$$

where the primes mark the differentiation performed with respect to the variable t. Then the general solutions for all the residual stresses can be present as [5, 17--19]:

$$\sigma_r = P_1' - \frac{c_1}{r^2} + 2k\ln r,$$
$$\sigma_\varphi = P_1' + \frac{c_1}{r^2} + 2k\left(1+\ln r\right), \qquad (30)$$

where P_1', c_1, k are the parameters bound with the boundary conditions and material constants. The initial two members of the presented general solution correspond to the functions that were previously used in (25). They describe only the mechanism of elastic stresses at the simplest approximation. The stresses, which appear in the region when $r < b$, are assumed to be “plastic”, as at quenching of amorphous materials plastic relaxation is possible.

Formulas (30) fully describe the formulated problem up to the radius *b* within the whole elastic region [18, pp. 112--113]. In the same manner, in the context of the theory of plastic stresses the solution region can be increased from the radius *b* to the dimensions where the validity criteria for the continuity model are violated (see the shaded area near zero in Fig. 7).

Let us analyze the additions to stresses in the "plastic" region ($r < b$) in the form which correlates with more detailed theory [19] (the Poisson ratio ν being ½ according to [17--19]). The analytic form of solution (30) sufficient for calculation of magnetic structure is known in the theory of plastic relaxation as the solution which makes allowance for the Tresca yield conditions (see [18, 19]). Let us present these functions in the following form:

$$\begin{aligned}
\sigma_{r(2)} &= 2K \times \ln\left(\frac{r}{b}\right),\\
\sigma_{\varphi(2)} &= 2K \times \left[1 + \ln\left(\frac{r}{b}\right)\right],\\
\sigma_{z(2)} &\approx \nu \times \left(\sigma_{\rho(2)} + \sigma_{\varphi(2)}\right) \approx 2K \times \left[1 + 2\ln\left(\frac{r}{b}\right)\right].
\end{aligned} \qquad (31)$$

Please note that one can add to formulas (30) and (31) some functions, weakly dependent on the variable with respect to radius [19], with no considerable contribution in our case of calculation of domain structure. Estimations of *K* and *b* based on experiment and physical grounds (see [5]) are as follows: the upper bound $b \le R_m/2$ and lower bound $K \sim 0{,}1P$.

We are suggested a model in which the residual stresses σ_r and σ_z (for brevity sake denoted as $\sigma_{r,z}$) in the CGCMNW strand monotonically decrease towards the strand center (see Fig. 7). This model differs from, say, the models in [20, 21] in boundary conditions that leads to qualitatively distinct behavior of the residual stresses (Fig. 7, curve γ). In [20, 21] the residual stresses $\sigma_{r,z}$ monotonically decrease to the strand--glass boundary, being not supported physically.

In the case of our model the decrease of $\sigma_{r,z}$ towards the center is caused by relaxation of stresses in the CGCMNW center. These plastic relaxations of elastic stresses can be explained by the fact that in the course of the strand cooling the temperature gradient between the center and periphery is hundreds degrees. The microwire surface is bounded with glass through a chemical energy bond that supersedes the energy of the residual stresses at the onset of the CGCMNW production. Only elastic relaxation of stresses mainly takes place till the internal radius *b*. The stresses which occur in the microwire region close to the strand center ($r < b$) is assumed "plastic", as at quenching of amorphous materials there can arise plastic relaxation. For illustrative purposes, there are presented some patterns of stresses in the amorphous strand and silicate glass (Fig. 7). Radial residual stresses are of interest for the domain model under consideration. All the residual stresses in the microwire strand are tensile (positive). The negative addition $\sigma_{r(2)}$, connected with relaxation (formulas (31)), is less than the positive (tensile) stress already existing in the metal strand (with respect to formulas (15) and (20)). For those $r << b$, where this addition becomes more than already existing residual stresses, the domain of applicability of our model is most likely violated. This region is "cut off" from our consideration (shaded area round zero in Fig. 7).

In the glass envelope the residual stresses are compressive (Fig. 7). According to the condition of equilibrium on the strand--glass surface, when the strand is extended, the glass is compressed. We also assume that after the glass envelope is taken away (or damaged) the radial stresses inside the strand can become compressive (say, in the inner area of the strand that is marked in Fig. 7 by the dashed line β). It becomes possible to determine more accurately the physical meaning of the parameter *b*.

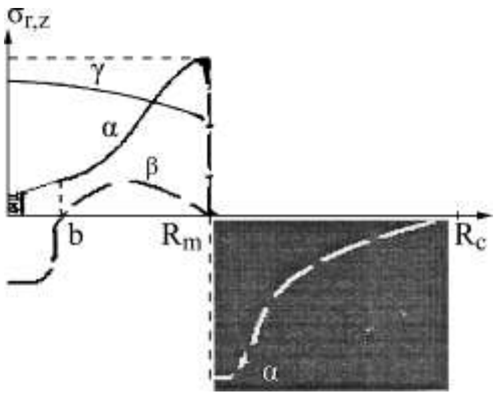

Fig. 7a. Qualitative pattern of the change of residual stresses $\sigma_{r,z}$ along the microwire radius. Curve α presents the initial state of these stresses in the metal strand and glass (in glass it is shown by black color and dashed line). In the metal strand these stresses are always tensile, i. e. positive. The residual stresses in the silicate glass always compress the glass envelope (i. e. they are negative). Inside the region of plastic relaxation there is shown a section (near the strand center) where the applicability of the continuum model is violated. Curve β is a hypothetic form of residual stresses in the metal strand after etch removal of the glass envelope or disruption of its bound with the metal strand. As an example, there are presented the qualitative results of the calculation of residual stresses according to [20, 21] (curve γ, which are different from our data.

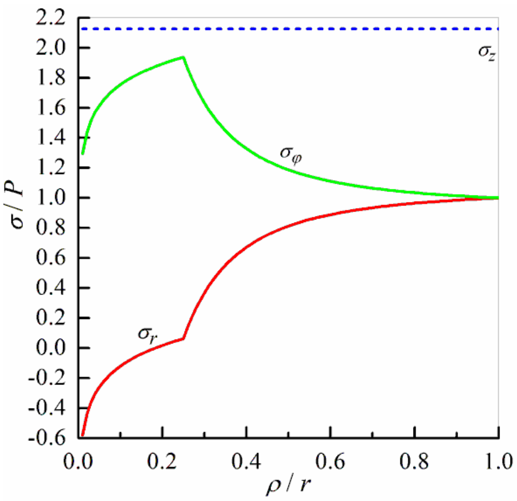

Fig. 7b. Results of calculations of dependences of distribution of pressure σ_r and σ_φ on microwire radius under formulas (25), (30), (31) are presented.

(At calculations it was accepted, that b=0.25r, k=0.4 and R/r=1.25. In the same drawing zero approach for calculation σ_z (the formula (15)) is resulted).

In conclusion we note the Poritzky--Hull--Burger model used here as a boundary condition to calculate residual stresses in CGCMNW is not the only possible. For instance, another pattern of calculation is proposed in [22]. Their obtained result differs from formulas (15). However, formulas (15) adequately explain the experiments concerning FMR and EFMR [23, 24], being inconsistent with the data in [22].

Appendix 2

An exchange energy member in the form of formula (3) is used with the aim to calculate the DW dimensions. The same formula is applied, say, in works [9, 25, 26], where the authors also considered the method of its derivation. As this formula is important we present the detailed derivation of it hereinafter.

Two nearest spins in a ferromagnetic, oriented at a slight angle relative to each other, increase the exchange interaction energy by

$$W_A \approx -\cos\varphi \approx -\left(1-\varphi^2/2\right). \qquad (32)$$

For the sake of simplicity, let us omit the interaction constant, that is proportional to the quantity A, as we are interested only in a functional dependence of this energy on the DW dimension, i. e. on δ. Hereafter we will make allowance only for an increment in the energy which makes its contribution at the variation of energy in order to find δ. For this reason the constants in (32) will be omitted.

The angle between vectors is proportional to the distance between spins a and the angle gradient. The average angle gradient is proportional to the angle-shift value which should be also divided by the DW dimension δ.

Thus, there is obtained the following intermediate quantity that is proportional to the quantity ($a^2/2\delta^2$).

With the aim to calculate the final result, it is necessary to multiply the sought quantity ~ ($a^2/2\delta^2$) by the number of rotation layers within the interval equal to the DW thickness, that is by the value equal to $\pi(\delta/a)$.

Finally, we obtain formula (3) with the omitted multiplier $\pi/2$ as not influencing the accuracy of the calculation.

The presented considerations are meaningful, if the dimensions of the DW are much more than the quantity a. In this case we pass to the continuity approximation in which a rapid rotation of two nearest spins by an angle of $\sim\pi$] is changed by a slow turn of the spin system which takes place over the length, equal to the DW dimension δ.

References

1) Lomaev G..V., Baranov S.A., Karimova G..V. On the possible mechanisms of switching for cast amorphous microwires. *Magnetic properties.* Ed. Lomaev G..V. Izhevsk GTU, 2011,. **4**, 43-47.

2) Lomaev G..V., Petrov M.Yu., Hodyrev A.V. Mathematical modeling of the process of switching for bistable ferromagnets. *Vestnik Udmurtskogo universiteta (phys.)* 2005, **4**, 195–202.

3) Lomaev G.V., Akhizina S.P., Vodenikov S.K. Two pulse forms for the magnetization jump in ferromagnets. *Defektoskopiya*, 1996, **12**, 54–59.

4) Lomaev G.V., Akhizina S.P., Glushkova T.E. Simulation of large jumps of Barkhausen. *Fiz. Met. Metalloved.* 1997, **84 (5)**, 461–465.

5) Baranov S.A. Magnetic models of cast amorphous microwires. *Surf Eng Appl Electrochem*. 2011, **47(4)**, 316–330.

6) Baranov S.A., Karimova G..V., Lomaev G..V. Domain wall movement in the cast amorphous microwire. *Surf Eng Appl Electrochem.* 2006, **42(2)**, 73–78.

7) Karimova G..V., Hodyrev A.V. Simulation fluctuations of start switching field in the glass coated amorphous microwires on the basis of Fe and Co. *Izvestia Instituta matematiki i informatiki*. Izhevsk. 2006, **2**, 173–176.

8) Hodyrev A.V. Simulation of stochastic resonance in bistable ferromagnets. *Abstracts of scientific conference "Control Theory and Mathematical Modeling"* 19 june. Izhevsk. UGU. 161–162.

9) Mishin D.D. Magnetic materials. M.: Vysshaia shkola, 1981. 335 c.

10) Gudoshnikov S.A., Grebenshchikov Yu.B., Ljubimov B.Ya., Palvanov P.S., Usov N.A., Ipatov M., Zhukov A., Gonzalez J. Ground state magnetization distribu-tion and characteristic width of head to head domain wall in Fe-rich amorphous microwire. *Phys. Status Solidi.* 2009, **A 206(4)**, 613–617.

11) Ivanov A.A., Lobov I.V., Vorobiev Yu. D. Some mechanisms of pinning of the domain wall in thin magnetic films. *Fiz. Met. Metalloved.* 1984, **58(1)**, 11–20.

12) Hilzinger H.R., Kronmüller H. Statistical theory of Bloch walls by randomly distributed defects. *JMMM*. 1975, **2(2-3)**, 11-17.

13) Kekalo I.B. Nanocrystalline soft magnetic materials. M.: MISiS, 2000. №1561. 250 c.

14) Soeda M., Takajo M., Yamasaki J. Large Barkhausen discontinuites of die – drawn FeSiB amorphous wire. *IEEE Tran. on Magnetics.* 1995, **31(6)**, 3877-3879.

15) Lyubimov, L.B. Spai metalla so steklom (Glass-Metal Seals) , Moscow: Energiya, 1968, pp. 131-142.

16) Baranov S.A. A three-layer model of an amorphous microwires. *Surf Eng Appl Electrochem*. 2010, 46(3), 271–275.

17) Amenzade, Yu.A., Teoriya uprugosti (The Theory of Elasticity), Moscow: Vysshaya Shkola, 1976. 272 p.

18) Godfri, D.E.P. Teoriya uprugosti i plastichnosti. (Theoretical Elasticity and Plasticity for Engineers), London: Thames and Hudson, 1965; Kiev: Budivel'nik, 1969.

19) Boley, B.A. and Weiner, J.H., Theory of Thermal Stresses. P. 2, New York, London: John Willej and sons, INS, 1960, pp. 470 – 550.

20) 20. Antonov A.S., Borisov V.T., Borisov O.V., Prokoshin A.F., Usov N.A. Residual Quenching Stresses in Glass-coated Amorphous Ferromagnetic Microwires. *J Phys D: Appl Phys*. 2000, **33**, 1161–1168.

21) Velazquez J., Vazquez M., Zhukov A.P. Magnetoelastic Anisotropy Distribution in Glass-coated Microwires. *J Mater Res*. 1996, **11(10)**, 2499–2505.

22) Zameatina, E.V., Petrzhik, M.I. Calculation of internal stresses generated during the manufacture of amorphous glass-coated microwires. *Deformatsia i razrushenie materialov.* 2010, **7,** 41–45.
23) Baranov S.A., Berzhansky V.N., Kokoz V.L., Zotov S.K., Larin V.S., Torkunov A.V. Ferromagnetic resonance in amorphous magnetic wires, *Fiz. Met. Metalloved.*, 1989, **67 (1)**, 73 – 78.
24) Baranov S.A., Zotov S.K., Larin V.S., Torkunov, A.V. Characteristics natural ferromagnetic resonance in amorphous microwires. *Fiz. Met. Metalloved.*, 1991, **69 (12)** 172 – 174.
25) Döring, W. Switch nucleation growth under the large Barkhausen jumps, *Usp.Fiz. Nauk*, 1939, **22**, **no.1(5),** 78-92.
26) Vonsovskii C.V. Magnetism. M. Nauka 1971. 797 p.