Vol 5, Issue 10, October 2025 www.ijesti.com E-ISSN: 2582-9734
International Journal of Engineering, Science, Technology and Innovation (IJESTI)

An In-Depth Examination of Metric Geometry and Metric Space Theory and Their Multidisciplinary Applications

Shishir Kumar Murmu
Assistant Professor, Ramananda Centenary College, Laulara, Purulia, West Bengal.
Email: murmushishir9@gmail.com

ABSTRACT

Mathematical disciplines that study spaces where distances between points may be measured include metric space theory and metric geometry. These theories lay out a rigorous framework for studying the shapes, sizes, and relationships of things in real and virtual worlds. Geometric and analytical concepts may be applied to large regions by starting with the basic concept of a metric, which is a function that defines the distance between points. An exhaustive review of metric space theory and metric geometry is given in this academic paper. The research goes beyond only mathematics to show how metric ideas are applied in many other areas, including computer science, physics, engineering, data analysis, economics, and biology.

Keywords: *Metric Geometry, Space Theory, Mathematics, Distance, Shape.*

I. INTRODUCTION

The mathematical foundation for a precise and expanded understanding of distance, shape, and structure is provided by metric space theory and metric geometry. They provide a unified language for post-Euclidean geometry, which allows us to look at spaces that aren't necessarily smooth or linear but do include distance. In an effort to formalize ideas of structure and distance independent from traditional geometric intuitions, mathematicians in the late 19th and early 20th centuries sought to establish metric geometry. In 1906, metric spaces were formally presented by Maurice Fréchet, revolutionizing the field. This opened up new possibilities for the study of geometry and topology in any location. The investigation of compactness, continuity, convergence, and connection within a coherent framework was made possible by this abstraction. Topology, functional analysis, differential geometry, and other branches of pure mathematics rely heavily on metric geometry and metric space theory, while computer science, physics, data science, and machine learning rely heavily on it as well.

Simply said, any two points in a metric space may be calculated by using a built-in function called a metric. The non-negativity, identity of indiscernible, symmetry, and triangle inequality are the four necessary axioms that this distance function must satisfy. These simple but powerful conditions allow distance to be formalized in many different contexts, from the well-known geometry of Euclidean space to very complicated mathematical frameworks. Open and closed sets, sequence convergence, and function continuity are all concepts that are governed by the topology that the metric generates on the space. Metric spaces allow us to explore both the local and global aspects of spaces, thereby connecting topology and geometry.

Metric geometry's development brought these ideas to a higher level by drawing attention to the quantitative and geometric aspects of metric spaces. Classical differential geometry places heavy reliance on smoothness and differentiable features, whereas metric geometry examines spaces where they may not be present. Understanding curvature, geodesics, and dimensional properties in a strictly metric setting is now the focus. Strong methods for investigating curvature restrictions, compactness theorems, and the large-scale geometry of spaces were developed by pioneering work of mathematicians such as Alexandrov, Gromov, and Busemann. These developments led to the definition of Gromov-Hausdorff convergence, which allows mathematicians to compare and analyze the limits of different metric spaces in a systematic way. In areas such as geometric group theory, where groups are investigated by examining the metric spaces on which they operate, this has far-reaching consequences.

When considering the concept of curvature in contexts other than smooth manifolds, metric geometry provides an appropriate framework. For instance, in cases when smoothness isn't applicable, Alexandrov's notion of curvature that is restricted above or below extends conventional notions of curvature. Equally important to modern geometric group theory is Gromov's theory of hyperbolic spaces, which has implications in computer science, combinatorics, and topology. Using these models, researchers may examine generalizations that stand in for intuitive geometric phenomena in very abstract settings, such as spaces of nonnegative curvature (CAT(0) spaces) and spaces of nonpositive curvature. The theory has also found applications in metric measure space analysis, optimal transport, and non-smooth Ricci curvature bounds in the last several decades. This has resulted in substantial connections between functional analysis, probability theory, and geometry.

Beyond this narrow focus, metric space theory provides a method for measuring structure, similarity, and proximity in a broad variety of contexts. Algorithms that cluster data, locate nearest neighbors, and decrease dimensionality are all based on the concept of a metric in computer science. Metrics such as the Euclidean distance, cosine similarity, and Wasserstein distance help us understand the relationships between data points in multi-dimensional domains. When developing loss functions, kernel methods, and representation learning models, metrics play a crucial role in machine learning as well. The formal description of spacetime forms and field configurations is given by metric spaces in physics. Concepts of metrics are useful in the social sciences and economics for analyzing preference landscapes, market distances, and utility-based decision-making frameworks. Metric geometry, then, is a means by which mathematical and scientific principles may be bridged, transforming theoretical frameworks into practical applications.

The flexibility and breadth of metric geometry are its greatest assets. Spaces containing singularities, discontinuities, or even structures resembling fractals can be studied using metric geometry. The geometry of Euclidean space is limited to perfectly smooth, inflexible objects. For instance, manifolds with singularities, networks, and forests may all be described in metric spaces. In their study of weighted metric spaces and spaces of low Nagata dimension, Basso and Sidler (2023) demonstrated methods for approximating these spaces with weighted trees, shedding light on how to understand complicated structures through simpler tree-like representations. Thanks to this capacity for abstract thought, we may investigate biological systems, mathematical physics, and digital network architectures with highly irregular topologies. The metric technique unifies several mathematical ideas into a single framework, which is one of its many advantages. You may discuss topics like compactness, continuity, and convergence without using differentiability or coordinates. Simply referring to distances will suffice for this purpose. This is why metric spaces are fundamental to analysis and topology. Linear operators and vector spaces with an infinite number of dimensions are studied in functional analysis on Banach and Hilbert spaces, which are full metric spaces with additional algebraic structure. One of the cornerstones of optimal transport theory is the Wasserstein metric, which is a major distance measure in probability theory. Geometric analysis has entered new frontiers thanks to the study of metric measure spaces, which are metric spaces endowed with measures. This has allowed geometric and analytic approaches to be extended to complicated and non-smooth situations.

Metric geometry's widespread use has also sparked fresh research. By facilitating the development of algorithms for object recognition, form analysis, and pattern matching, metric geometry is a useful tool in computer vision and image processing. You may assess the similarity of features and representations by embedding data in metric spaces in machine learning, particularly deep learning. Metrics play a crucial role in robotics and motion planning for determining the distance between various configurations and paths in intricate scenarios. Metric embeddings and distortion limits are helpful tools in theoretical computer science for determining algorithm efficiency and graph structure. Measurements of semantic distances and descriptions of mental knowledge constructions are both made possible by metric concepts in linguistics and cognitive science. These instances illustrate the pervasiveness of metric geometry in modern scientific reasoning.

The evolution of metric geometry marks a turning point in human understanding, marking the passage from concrete to generalized concepts. There is a philosophical movement in mathematics towards rigor, structural comprehension, and generalizability, and this is mirrored in the departure from Riemannian and Euclidean perspectives in favor of metric formulations. During the early 20th century, Fréchet brought metric spaces to topology, which had a profound impact. Subsequent work by Hausdorff, Urysohn, and others formalized concepts like as completeness, compactness, and separability.

The geometric study of generic metric spaces became a prominent area of study in the 20th century as a result of the merging of metric and geometric notions, particularly in the works of Alexandrov and Gromov. Still in the present day, data science, computational geometry, and mathematical physics are driving changes to metric geometry. Its continued use and adaptability are demonstrated by this.

II. REVIEW OF LITERATURE

Leinster and M. Shulman (2021) Enhanced categories, such as metric spaces as $[0,\infty)$-enriched categories, possess the numerical invariant known as magnitude. We prove that magnitude may be reduced to a theory of enriched category homology, called magnitude homology (a specific kind of Hochschild homology), in many instances, where magnitude is the graded Euler characteristic. The generalization of the Hepworth-Willerton magnitude homology of graphs to magnitude homology of metric spaces allows the detection of geometric information like convexity.

N. Khan (2019) Each pair of elements in a set can be defined by a metric, also called a distance function. The bilinear forms of metric tensors, which may be described as the tangent vectors of a differentiable manifold onto a scalar, are the primary sources of metrics in differential geometry. The set X and the function d, sometimes known as the "distance function" and represented as d(x, y), form a metric space.

Bau, Sheng & Beardon, Alan. (2013) Consider the metric space (X,d). Each point x in X may be uniquely defined by the distances d(x,a), where an is a member of A, and therefore, a subset A of X resolves X. For each given set A with cardinality k, the metric dimension of X is the smallest number d such that X is resolved. We know a lot about the metric dimension for graphs where X is the set of all vertices, but for generic metric spaces, not much. For generic metric spaces, we present here a few elementary findings.

Vuorinen, Matti. (2011) For the last 30 years, hyperbolic type metrics have also found widespread use in contemporary mapping theory, particularly for studying quasiregular and quasiconformal maps in euclidean n-space. In this paper we review a number of metrics which are connected to contemporary mapping theory in some manner and highlight several unanswered questions about the geometry of these metrics.

M. W. Meckes (2010) In finite metric spaces, magnitude is a numerical invariant that was developed not long ago by T. Leinster; it is roughly equivalent to the cardinality of finite sets or the Euler characteristic of topological spaces. Multiple separate extensions to infinite metric spaces have been made a priori. In this work the theory of positive definite metric spaces is advanced, a special case of metric spaces where magnitude is less universal and more tractable. The classical feature of negative type for a metric space is known to hold for many relevant types of spaces; positive definiteness is an extension of this. For compact positive definite metric spaces, it is shown that all the given definitions of magnitude are equivalent, and further findings on the behavior of magnitude in relation to these spaces are also proven.

III. METRIC GEOMETRY

An essential part of mathematics, metric geometry uses mathematical precision to investigate sizes, forms, and other dimensions. As a result, students have a more thorough grasp of the geometric world around them and are able to delve more deeply into the abstract and practical parts of spaces and distances.

The study of space-structure relationships as a function of distance is central to metric geometry. Metric geometry places an emphasis on distance as the primary unit of measurement, in contrast to conventional geometry that may give more weight to angles and relative positions. This area of mathematics studies the relationship between distance and the definition, interaction, and behavior of forms in space and when transformed.

The study of geodesics, or the shortest path between two locations, or the behavior of circles in spaces other than Euclidean coordinates are two examples of topics covered in metric geometry. Similarities and discrepancies between common Euclidean spaces and exotic places, such as hyperbolic or spherical geometries, are brought to light by these explorations.

Key Principles of Metric Geometry

The foundational ideas of metric geometry set it apart from other areas of mathematics. To fully appreciate metric geometry and its many uses, one must have a firm knowledge of these concepts.

- **Distance Measurement**: The metric, a system for calculating distances between points, is fundamental to metric geometry.
- **Invariance under Isometry**: Metric qualities are unaffected by distance-preserving transformations like translations, rotations, and reflections.
- **Geodesics**: An essential idea in metric geometry is the study of geodesics, which are the shortest pathways between any two locations.
- **Triangles**: In order to grasp metric spaces, one must be familiar with triangle characteristics, such as the well-known triangle inequality.

Consider the methods used to estimate distances on Earth's surface to put these ideas into context. When two locations are located on a spherical (like Earth) surface, the shortest route between them is an arc, not a straight line. Geodesics are fundamental to metric geometry, and here is an example of one. It is via these kinds of instances that the similarities between metric geometry and the real and built environments become apparent.

Real-Life Applications of Metric Geometry

From navigation and space travel to building and engineering, metric geometry is ubiquitous in modern life. Its fundamental ideas lead to technological and aesthetic breakthroughs and the resolution of difficult challenges. Here are a few examples of real-world applications of metric geometry:

Architecture and Urban Planning

Metric geometry is a powerful tool for architects who strive to create visually beautiful and useful public spaces and structures. Accessibility and navigability of structures are guaranteed by taking distances and geometric relations into account.

Geography and Cartography

Metric geometry's notion of vast circles aids cartographers in creating precise maps. The use of geodesics and knowledge of the Earth's curvature enables the visualization of the shortest pathways between geographical sites.

Astronomy and Space Exploration

To find their way through the infinite expanse of space, astronomers rely on metric geometry to determine distances between stars and planets.

Solving Problems with Metric Geometry

Metric geometry provides a strong foundation for addressing geometric issues and also has practical applications. Problems can be approached and solved by focusing on distances and spatial interactions by using the features of metric spaces. Using metric geometry in this way helps in solving problems:

Understanding The Properties of Shapes and Spaces

One may intuitively ascertain the qualities and relations of geometric forms by applying ideas from metric geometry, like geodesics and the triangle inequality.

Optimizing Paths and Layouts

Finding the most efficient pathways and arrangements is made possible by metric geometry, which is useful in many contexts, such as city street layout and electrical wiring routing on circuit boards.

Modelling Complex Systems

The internet and weather systems are only two examples of the many complex systems that may be better understood using metric geometry, which in turn allows for the creation of more robust and efficient networks.

IV. METRIC SPACE

The mathematical concept of metric space is a method for determining the lengths and distances between known items in a collection. When joined together, any two or more items that are otherwise considered separate might be considered part of a set. From concrete objects like buildings or people to more nebulous concepts like numbers, sets may contain everything. An element is any component that makes up a set.

A city, for example, may be thought of as a set. After then, you might think of every structure in the city as a part of the set. A metric space is a set in which all the distances between each pair of elements have been determined. A metric is the distance between any two elements in a set, as in the previous example, between two buildings in the city.

Every pair of elements can have a positive real value (any integer larger than zero) assigned to these distances according to calculations in metric space.

It was French mathematician Maurice Fréchet who initially defined metric space. Jacques and Zoé Fréchet had four children, making Fréchet the fourth child of six. Before becoming the principal of a Protestant school, his father had a distinguished career in education; he was running an orphanage when his son was born. The number theory scholar Jacques Hadamard was Fréchet's professor. In the end, Hadamard oversaw the writing of Fréchet's dissertation for his doctorate after recognizing his mathematical abilities and personally tutoring him.

Need To Study Metric Space

A theoretical subfield of mathematics known as functional analysis developed some 80 years ago with its foundations in classical analysis. Even now, its methods and results are relevant to many branches of mathematics and their applications in the real world. Linear integral equations, calculus of variations, linear ordinary and partial differential equations, linear algebra, approximation theory, and calculus of variations all share many traits and characteristics, as mathematicians have seen.

Making use of this fact enabled a unified approach to these difficulties, accomplished by leaving out superfluous details. Consequently, zeroing down on the most important details is the advantage of taking such an abstract approach. Instead of defining the components' properties, an abstract method often begins with a collection of elements that meet specific axioms. A mathematical framework with an abstractly stated theory can be obtained via the axiomatic technique. It is possible to apply the general theorems to specific sets that fulfill those axioms in the future.

If we take algebra as an example, we see this technique used with fields, rings, and groups. Because of their similarity to calculus's real line $\wp$, metric spaces also serve as a basis for functional analysis. Actually, they build on top of $\mathbb{R}$ and were presented to provide the groundwork for a unified strategy to major challenges in several fields of study.

Types of Metric Space

Complete Metric Spaces

For a particular metric space to be considered complete, all Cauchy sequences must converge there. In other words, if d(x_n ,y_m) → 0 there exists a y∈ M that fulfills the condition when both and independently approach infinity d(x_n ,y) → 0.

As example: d(x,y)=│x-y│, in terms of its absolute worth, space is incomplete.

Compact Metric Spaces

A metric space is considered compact when each sequence contains a unique subsequence that converges to a specific point. Hence, it is also known as sequential compactness; yet, when applied to metric spaces, it resembles countable compactness in topological theory and is subsequently supplied by open cover.

Locally Compact Spaces

The condition that all points in a metric space that are compact neighbors are points in the same space is what causes every point to become locally compact. It is not required that infinite-dimensional spaces, especially Banach spaces, be locally compact; however, the same does not hold for the Euclidean spaces. Space is said to be appropriate when all closed balls in a particular metric space become compact. Even if proper spaces can be locally compact in general, it doesn't mean the inverse is always true.

Uniform Continuity for Metric Spaces

The function T: X→Y becomes uniformly continuous for any real integer when there are two metric spaces (X, k_1) and (Y, k_2). ε > 0 exists some real δ> 0 and k_1(x,y) < δ, there is k_2(T(x),T(y))< ε for x, y∈ X.

If X is compact, then the opposite of every uniform continuous mapping produced by any function T: X→Y is also continuous.

Lipschitzcontinuous Map

If the following condition is met and the real number R> 0, then the mapping T: X→Y is lipschitz continuous:

d_2(T(x), T(y)) ≤ Rd_1(x, y) when x, y∈ X.

On the one hand, lipschitz continuous mapping can never be satisfied as truth, and on the other, it always becomes uniform continuous.

The contraction function T is defined when R is less than 1.

If X is complete, then Y must be as well. Here, the function Tcontains a unique fixed point if T becomes a contraction map.

Convergence of Metric Spaces

Let the metric space be (X,d) then the sequence $\{x_n\}$ having points in X converges to x∈ X when for ε > 0, there exist an integer N satisfies the condition:

d(x_n, x)< ε when n>N

Here, we know that the limit point of that sequence is point x in X. Also, every convergent sequence has some kind of bound.

Applications of Metric Space

In contemporary mathematics and other mathematical branches of research, metric spaces play a crucial role. When formally measuring distances between objects, metric spaces are useful. Structure, convergence, and continuity may be studied in both theoretical and practical contexts by mathematicians and scientists.

Mathematics

Topology, analysis, and geometry are all branches of pure mathematics that rely on metric spaces. They make it possible to define notions like limits, continuity, open and closed sets, and calculus and functional analysis rigorously. Full metric spaces offer the setting for the formulation of several important theorems, such the Banach Fixed Point Theorem, which ensures that specific self-maps have fixed points and that these points are unique. Spaces are categorized in topology according to their geometric qualities by the notion of metric equivalence, which finds use in the areas of compactness, connectedness, and uniform continuity.

Computer Science

When it comes to data analysis, machine learning, and algorithm creation, metric spaces are crucial in computer science. Distance metrics such as the cosine, Hamming, Manhattan, and Euclidean can be used to quantify the similarity between two data points. Dimensionality reduction techniques, clustering algorithms (like k-means), and nearest neighbor searches all rely on these measures. When it comes to image processing, voice recognition, and natural language processing, metric-based models are invaluable tools for AI systems. They help with pattern matching, feature extraction, and similarity identification.

Physics and Engineering

In engineering and physics, metric spaces are helpful for explaining the geometry of space-time and the behavior of physical systems. To illustrate the curvature of space-time and the effects of mass and energy on gravitational fields, the concept of a metric tensor is used in general relativity. Control theory in mechanical and electrical engineering makes use of metric concepts; in this area, quantifying the distance between signals or pathways is crucial to ensuring system stability and minimizing mistakes.

Economics and Social Sciences

Metric spaces facilitate quantitative examination of preferences, choices, and risk in the social sciences and economics. To help with optimization and forecasting, econometric models use distance functions to measure how similar or different economic data is. Metric methods are useful in the fields of psychology and sociology for describing and measuring ideas, behaviors, and attitudes within complex theoretical frameworks.

Biological Sciences and Medicine

Bioinformatics makes use of metric space theory in the medical and biological sciences; in this field, distance metrics between genetic sequences are used for sequence alignment and phylogenetic tree construction. Aside from that, it's useful in diagnostics and medical imaging since metrics may be used to compare and organize trends in biological data.

Network Theory and Geography

Network theorists and geographers use metric spaces as a lens through which to view spatial relationships and connectedness. The shortest distance between nodes may be found using graph metrics. If we want to know how to get things done, where to locate facilities, and how transportation and communication network’s function, we need to know this.

V. CASE STUDIES AND EXAMPLES OF METRIC GEOMETRY AND METRIC SPACE THEORY APPLICATIONS

Image Recognition

When it comes to image recognition problems, metric space theory is a lifesaver when it comes to similarity metrics. Feature or descriptor distance metrics in images are defined in metric spaces for this application. To find how similar two photos are, we measure how far apart their feature representations are in metric space. Euclidean distance, cosine similarity, and Mahalanobis distance are some distance metrics that are utilized in picture recognition.

Recommender Systems

Recommender systems employ metric space theory to provide consumers with information or goods that they are likely to like. Collaborative filtering methods that are based on similarity employ metric spaces to determine how similar two persons or objects are. To provide informed suggestions, the metric space's distance measures are useful for locating related persons or objects. Cosine similarity, Pearson correlation coefficient, and Jaccard similarity are among distance metrics used in recommender systems.

Geographic Information Systems (GIS)

In order to analyze spaces and calculate distances, GIS applications frequently employ metric geometry and metric space theory. To find the distance between two points in the real world, scientists utilize metrics like network distance, geodesic distance, or even the more traditional Euclidean distance. Using these measures, algorithms for navigation and routing, site selection, proximity analysis, and geographic information system (GIS) spatial grouping are made easier.

Protein Structure Analysis

Protein structures are analyzed using metric space theory to learn about folding, stability, and function. A high-dimensional metric space is used to depict protein structures. Protein structural similarity and dissimilarity may be measured using distance measures like root mean square deviation (RMSD). Aligning and clustering protein structures, as well as predicting protein-protein interactions, are all helped by these measures.

VI. CONCLUSION

A flexible vocabulary for describing and solving complex problems in science, technology, and engineering is provided by the framework of metric spaces, which also improves understanding of fundamental mathematical concepts like compactness, convergence, and continuity. By offering a precise definition of distance, metric spaces allow for the systematic analysis of the patterns, structures, and linkages that support data-driven systems, computer processes, and physical occurrences. Topology, functional analysis, AI, physics, and bioinformatics are just a few of the many areas that make use of metric theory, making it an essential tool in both academic and professional circles. Studying metric spaces ultimately reveals the deep connections between arithmetic and other fields. Additionally, it demonstrates how modern civilization may benefit from abstract thinking by developing powerful instruments for creativity, discovery, and knowledge.

REFERENCES

1. G. Basso and H. Sidler, "Approximating spaces of Nagata dimension zero by weighted trees," *Illinois Journal of Mathematics*, vol. 67, no. 1, pp. 45–72, 2023.
2. T. Leinster and M. Shulman, "Magnitude homology of enriched categories and metric spaces," *Algebraic & Geometric Topology*, vol. 21, no. 5, pp. 2175–2221, 2021.
3. W. F. Pfeffer, "Flat forms in metric spaces," *Journal of the London Mathematical Society (II Series)*, vol. 104, no. 5, pp. 2402–2432, 2021.
4. N. Khan, "Metric space, applications and its properties," *International Journal of Scientific Research and Reviews*, vol. 8, p. 5, 2019.
5. N. A. Secelean, S. Mathew, and D. Wardowski, "New fixed point results in quasi-metric spaces and applications in fractals theory," *Advances in Difference Equations*, vol. 2019, pp. 1–23, 2019.
6. P. D. Andreev, "Normed space structure on a Busemann G-space of cone type," *Mathematical Notes*, vol. 101, no. 2, pp. 193–202, 2017.
7. B. Miesch, "The Cartan–Hadamard theorem for metric spaces with local geodesic bicombings," *Enseignement Mathématique (2)*, vol. 63, no. 1–2, pp. 233–247, 2017.
8. P. D. Andreev, "Proof of the Busemann conjecture for G-spaces of nonpositive curvature," *St. Petersburg Mathematical Journal*, vol. 26, no. 2, pp. 193–206, 2015.
9. M. Bačák, B. Hua, J. Jost, M. Kell, and A. Schikorra, "A notion of nonpositive curvature for general metric spaces," *Differential Geometry and its Applications*, vol. 38, no. 1, pp. 22–32, 2015.

10. V. Gupta, V. Bhardwaj, and R. Deep, “Some fixed-point results for A-contractions in 2-metric spaces and their applications,” *Miskolc Mathematical Notes*, vol. 16, no. 2, pp. 679–694, 2015.
11. L. Ambrosio, N. Gigli, and G. Savaré, “Calculus and heat flow in metric measure spaces and applications to spaces with Ricci bounds from below,” *Inventiones Mathematicae*, vol. 195, no. 2, pp. 289–391, 2014.
12. U. Lang, “Injective hulls of certain discrete metric spaces and groups,” *Journal of Topology and Analysis*, vol. 5, no. 3, pp. 297–331, 2013.
13. M. Çınar, M. Karakaş, and M. Et, “Some geometric properties of the metric space,” *Journal of Inequalities and Applications*, vol. 2013, no. 1, pp. 1–7, 2013.
14. S. Bau and A. Beardon, “The metric dimension of metric spaces,” *Computational Methods and Function Theory*, vol. 13, no. 2, pp. 1–7, 2013.
15. U. Lang, “Injective hulls of certain discrete metric spaces and groups,” *Journal of Topology and Analysis*, vol. 5, no. 3, pp. 297–331, 2013.
16. M. Vuorinen, “Geometry of metrics,” *The Journal of Analysis*, vol. 2, no. 1, pp. 1–27, 2011.
17. B. K. Lahiri, P. Das, and L. K. Dey, “Cantor’s theorem in 2-metric spaces and its applications to fixed point problems,” *Taiwanese Journal of Mathematics*, vol. 15, no. 1, pp. 337–352, 2011.
18. M. W. Meckes, “Positive definite metric spaces,” *Positivity*, vol. 17, no. 3, pp. 733–757, 2010.
19. K.-T. Sturm, “On the geometry of metric measure spaces. I,” *Acta Mathematica*, vol. 196, no. 1, pp. 65–131, 2006.
20. S. Wenger, “Isoperimetric inequalities of Euclidean type in metric spaces,” *Geometric and Functional Analysis*, vol. 15, no. 2, pp. 534–554, 2005.
21. V. Gregori and A. Sapena, “On fixed-point theorems in fuzzy metric spaces,” *Fuzzy Sets and Systems*, vol. 125, no. 2, pp. 245–252, 2002.
22. U. Lang and C. Plaut, “Bilipschitz embeddings of metric spaces into space forms,” *Geometriae Dedicata*, vol. 87, no. 1–3, pp. 285–307, 2001.
23. S. V. R. Naidu, “Some fixed-point theorems in metric and 2-metric spaces,” *International Journal of Mathematics and Mathematical Sciences*, vol. 28, no. 11, pp. 625–636, 2001.
24. L. Ambrosio and B. Kirchheim, “Currents in metric spaces,” *Acta Mathematica*, vol. 185, no. 1, pp. 1–80, 2000.