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Existence and Approximation Results for Hybrid Functional Differential Equations Using Dhage Iteration Methods

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ABSTRACT

The research focuses on fixed-point theory within partially ordered Banach spaces, emphasising Dhage's iterative approach. The primary methodology integrates partial Lipschitz criteria, nonlinear contraction principles, and compactness attributes to demonstrate the existence of solutions given appropriate assumptions. By converting the specified differential equations into corresponding operator equations, the examination isolates the nonlinear elements and enables the implementation of a hybrid fixed-point principle. The research additionally investigates the convergence of sequential approximations and demonstrates how a monotonic iterative series might facilitate the attainment of a solution. A significant aspect of this methodology is that it does not necessitate the preliminary establishment of both upper and lower solutions along with the corresponding monotonic inequalities. The findings offer a valuable structure for examining nonlinear functional equations where the current condition of a system may be influenced by its historical actions.

Keywords: *Hybrid, Dhage Iteration, Fixed-Point Theory, Banach Spaces, Nonlinear.*

I. INTRODUCTION

In mathematical analysis, hybrid functional differential equations are commonly employed to characterise systems whose behaviour is affected by both their current state and their historical values. Memory, delayed reactions, and nonlinear interactions are some of the real-world phenomena that these equations may capture. While many dynamic systems are studied using classic differential equations, hybrid functional differential equations allow for the description of more complicated interactions. These equations depict the behaviour of a system under different situations by combining distinct functional and differential components. In order to prove that solutions exist and to find the circumstances under which solutions can be produced, their mathematical characteristics are studied. Here is one way to represent a generic functional differential equation:

$dx(t)/dt = F(t, x_t), t \in [0, T],$

where x(t) represents the unknown function, F is a given functional operator, and x_t denotes the history of the solution up to time t. For a fixed delay $\tau > 0$, the history function may be defined by $x_t(\theta) = x(t + \theta)$, where $\theta \in [-\tau, 0]$. With this expression, the current rate of change may be determined by looking at the solution's values from the past.

A key difference between ordinary differential equations and functional differential equations is the fact that the latter rely on values of the unknown function at various points in time. A population's present and historical sizes, for instance, may affect the pace of change within that group. In engineering systems, control mechanisms, and biological processes, similar correlations are recognized. Delay terms and other functional expressions are used theoretically to depict such relationships. One way to express a basic delay differential equation is as

$x'(t) = ax(t) + bx(t - \tau),$

where τ stands for the time delay and a and b are constants. The mathematical formulation of hybrid functional differential equations may also integrate extra nonlinear components. The form of the equation determines which functional connections may be included, such as products, quotients, integral terms, and others. Consequently, more intricate mathematical questions may be posed, yet other challenges may arise when attempting to study their answers.

The word "hybrid" is often used to describe an equation that has many mathematical components. Multiple functional expressions, including delayed arguments, integral operators, linear and nonlinear terms, and others, may coexist in a single mathematical model, depending on the formulation. This is one way to describe a hybrid functional differential equation:

$d/dt\ [x(t)/g(t, x(t))] = f(t, x(t), x(\alpha(t)))$,

subject to an initial condition such as

$x(0) = x_0$,

If f and g are specified functions, α(t) is a functional variable, and x_0 is the designated starting condition. The specific structure of a hybrid equation is contingent upon the issue at hand. The resolution is affected by the interplay between the differential operator and the nonlinear functional elements. The examination may be challenging when these elements fail to meet the criteria stipulated by traditional existence and uniqueness theorems. In these circumstances, fixed-point theory offers valuable mathematical instruments for demonstrating the existence of solutions.

A significant technique used in this domain is the Dhage iteration method, grounded on fixed-point theory and nonlinear functional analysis. This approach transforms intricate nonlinear equations into operator equations, allowing for an analysis of the characteristics of the resultant operators. The initial issue may be restructured into the format.

$x = A(x)B(x)$,

where A and B are appropriately designated operators inside a relevant function space. The precise operator representation is contingent upon the configuration of the initial equation. The challenge of identifying a solution is therefore transformed into the challenge of locating a fixed point of the corresponding operator expression. Provided that certain conditions are met, one may demonstrate the existence of a fixed point by using a relevant fixed-point theorem. The resultant fixed point subsequently yields a resolution to the initial equation.

The Dhage iteration principle, sometimes referred to as DIP or DIM, has been formulated in the realm of nonlinear analysis, especially concerning nonlinear differential and integral equations. Its relevance has also been examined in fractional differential equations and many nonlinear issues. A conventional hybrid functional differential equation may be abstractly expressed as

$x(t) = A(x)(t)B(x)(t)$,

for some appropriate space of continuous functions, A and B are nonlinear operators. After that, you can build an iterative sequence by

$x_{n+1}(t) = A(x_n)(t)B(x_n)(t)$, n = 0, 1, 2, ...

subject to the iteration being suitable for the operators being considered. The sequence can be shown to converge if the necessary conditions are met, and its limit can be found as an equational solution. It is not possible to assume the particular convergence conditions for every nonlinear operator equation; rather, they must be checked for the chosen Dhage-type theorem.

Nonlinear issues that are difficult to solve using traditional methods can be better understood with the help of the Dhage iteration approach. If the equation can be reformulated as a set of operators having distinct mathematical characteristics, then some restrictive assumptions can be loosened. The development of these approaches incorporates important principles from fixed-point theory, Banach's contraction principle, and nonlinear functional analysis. In particular, according to the Banach contraction principle, if an operator T on a full metric space fulfils

$d(Tx, Ty) \leq k\, d(x, y), 0 \leq k < 1,$

For all x, y in the space, T has a unique fixed point. This concept lays the foundation for fixed-point analysis, however, Dhage-type methodologies can also use alternative fixed-point results involving other operators depending on the assumptions of the problem. Continuity, boundedness, compactness and prerequisites for the corresponding iterative procedure are given special emphasis. The existence of solutions is considered as an important issue in the study of hybrid functional differential equations. Before attempting to approximate a solution or analyze its properties we must establish existence of at least one solution. This requirement is met by converting the original differential equation into an equivalent operator equation in an appropriate function space. For instance, if the equation is given as an initial value problem,

$x'(t) = f(t, x(t)), x(0) = x_0,$

it may be expressed in integral form as

$x(t) = x_0 + \int_0^t f(s, x(s))\, ds.$

This transformation offers an operator formulation that can be investigated using fixed-point theory. The corresponding integral representation for hybrid equations may include supplementary functional or nonlinear terms. In order to apply an appropriate existence theorem, conditions of continuity, boundedness, Lipschitz-type properties, and the behaviour of nonlinear operators are examined.

Another critical component of Dhage iteration methods is the approximation of solutions. Then one can construct an iterative procedure to generate a sequence of functions which converges to a solution of the original equation. If the sequence $\{x_n\}$ converges uniformly to a function x*, then

$\lim_{n\to\infty} ||x_n - x^*||_\infty = 0,$

where the uniform norm is defined by

$||x||_\infty = \sup_{t\in[0,T]} |x(t)|.$

When the associated operator is continuous and the iteration satisfies the required conditions, the limit may be shown to satisfy the operator equation. But one cannot assume convergence without a mathematical justification. The properties of the operators, the structure of the iteration scheme and the assumptions imposed on the equation must be examined carefully. The existence of a solution, its uniqueness, and the convergence of an iterative sequence are therefore treated as distinct mathematical questions.

Hybrid functional differential equations studied through Dhage iteration methods represent an important connection between differential equations, nonlinear analysis, and fixed-point theory. This allows for the study of equations with complicated functional dependencies in a systematic mathematical framework. The operator formulation allows to treat separately each of the term of the equation, and appropriate fixed point theorems provide the basis for the existence of solutions. The effectiveness of the method depends on the assumptions made and the mathematical properties satisfied by the operators.

II. REVIEW OF LITERATURE

Lachouri, Adel et al., (2021). Using a measure of noncompactness in conjunction with the fixed point theorem of M?nch, we investigate whether solutions exist for nonlinear sequential Caputo and Caputo-Hadamard fractional differential equations that have three-point boundary conditions. The usefulness of the theoretical findings is shown by an example.

Dhage, Bapurao. (2020). A hybrid fixed point theorem of the author is used to show an existence result for perturbed abstract measure differential equations under combined generalised Lipschitz and Carathéodory conditions. Additionally, the author's hybrid fixed point theorem on ordered Banach spaces is used to demonstrate the existence of the extremal solutions under certain monotonicity criteria. Among our existence findings are those of the author and, as special examples, those that are subject to lesser continuity conditions.

Dhage, Bapurao. (2019). Through the development of an algorithm, we demonstrate the presence and approximate solution to a nonlinear starting value issue involving second-order neutral-type hybrid functional differential equations. The current hybrid fixed point concept of author is based on the Dhage iteration technique, which is the major resultant. The paper's abstract and its assumptions are backed up with an example. Let $J = [-r, T]$ and think about the closed and bounded intervals $I\ 0 = [-r, 0]$ and $I = [0, T]$ in R, where $r > 0$ and $T > 0$. In this context, "C" refers to the space of continuous real-valued functions defined on I 0, and "R" is the set it contains. The norm is applied to the space C. C is defined as the set of all real numbers x, where $x(\theta)$ is the mean of all real numbers x from 0 to θ. Section 1.1 The history space of the relevant functional differential equation is C, which is obviously a Banach space with this supremum norm. For every continuous function $x: J \to R$ and every $t \in I$, we represent the element of the space C as $x\ t\ (\theta) = x(t + \theta)$, where $-r \leq \theta < 0$. This is stated in equation 1.2.

Dhage, Bapurao & Otrocol, Diana. (2018). We consider the existence and approximation of solutions to the initial value issue of first order nonlinear differential equations with maxima in this work. The author's recent hybrid fixed point theorem in a partly ordered normed linear space incorporates the Dhage iteration approach, which is the major conclusion. Finally, we provide an example to show how the paper's abstract conclusions may be used and how the hypotheses are supported.

Dhage, Bapurao. (2017). This study presents an explanation of the power of a novel iteration approach in applications and discusses two nonlinear hybrid functional differential equations that include a delay. Specifically, by building an algorithm, we demonstrate the uniqueness and existence of initial value problems involving first-order nonlinear hybrid functional differential equations and their approximate solutions. In a partly ordered normed linear space, the primary outcomes depend on the Dhage iteration approach as expressed in the current hybrid fixed point concept of the paper's abstract findings and assumptions are backed up by examples.

Dhage, Bapurao et al., (2016). The authors of this study demonstrate, by means of the operator theoretic approach in a partly ordered metric space, that the solutions to initial value problems involving nonlinear fractional differential equations subject to nonlocal conditions exist, are unique, and can be approximated. In a partly ordered normed linear space, the key findings are based on the Dhage iteration principle, which is realised in the new hybrid fixed point theorem of Dhage (2014). Under weaker mixed partial continuity and partial Lipschitz conditions, the approximation of the solutions of the nonlinear fractional differential equations that are investigated are found. We also provide a numerical example to demonstrate our results and theories.

Dhage, Bapurao et al., (2016). An initial and periodic boundary value issue of nonlinear second order ordinary differential equations is proved, along with strategies for their existence and approximation, in this study. The essential findings are based on the Dhage iteration principle, which is encapsulated in a new hybrid fixed point theorem of Dhage (2013) for partially ordered normed linear spaces. The equations under consideration have their numerical solutions found under weaker mixed partial continuity and partial Lipschitz conditions. Several numerical examples also serve to demonstrate our theories and findings.

III. AUXILIARY RESULTS

Throughout this section, unless otherwise mentioned, let (E, ≤, ||. ||) denote a partially ordered normed linear space. Two elements x and y in E are said to be comparable if either the relation $x \preceq y$ or $y \preceq x$ holds. A non-empty subset C of E is called a chain or totally ordered if all the elements of C are comparable. It is known that E is regular if {xn} is a non-decreasing (resp. non-increasing) sequence in E and xn → x∗ as n → ∞, then $x_n \preceq x^*$ (resp. $x_n \succeq x^*$) for all n ∈ N. The conditions guaranteeing the regularity of E may be found in Guo and Lakshmikantam and the references therein.

We need the following definitions appeared in the development of rest of the study.

A mapping T : E → E is called isotone or nondecreasing if it preserves the order relation , that is, if $x \preceq y$ implies $\mathcal{T}x \preceq \mathcal{T}y$ for all x, y ∈ E. Similarly, T is called nonincreasing if $x \preceq y$ implies $\mathcal{T}x \succeq \mathcal{T}y$ for all x, y ∈ E. Finally, T is called monotonic or simply monotone if it is either nondecreasing or nonincreasing on E. A mapping T : E → E is called partially continuous at a point a ∈ E if for > 0 there exists a δ > 0 such that $\|\mathcal{T}x - \mathcal{T}a\| < \epsilon$ whenever x is comparable to a and k x – a k < δ. If at every point on E, T is partially continuous, we say that it is partially continuous on E. It follows that for every chain C in E, T is continuous if and only if it is partially continuous on E. If each chain C in a non-empty subset S of the partially ordered Banach space E is bounded, then S is said to be partially bounded.

When an operator T on a partially normed linear space E into itself is defined as a partially bounded subset of E, we say that T is partially bounded. If every chain C in T (E) is bounded by a unique constant, we say that T is uniformly partially bounded. Partial compactness is defined as the property that every non-empty subset S of a partially ordered Banach space E has chains C that are relatively compact subsets of E.

A mapping T : E → E is called partially compact if T (E) is a partially relatively compact subset of E. Any operator T on E that is both partially compact and uniformly partially bounded is referred to as uniformly partially compact. When T is a partially relatively compact subset of E for any bounded subset S of E, we say that T is partially totally bounded. "Partially completely continuous" describes a function T on E if it is both partially continuous and partially totally bounded.

Remark

Let T be an operator on E that does not decrease into itself. Assuming that for every chain C in E, T (C) is a bounded or relatively compact subset of E, we say that T is partially bounded or partially compact.

Definition

If {x n} is a monotone sequence in E and a subsequence {x n k} converges to x∏, then the original sequence {x n} converges to x∏ as well, and the order relation and metric d on a non-empty set E are said to be D-compatible.

Similarly, given a partially ordered normed linear space $(E, \preceq, \|\cdot\|)$, the order relation and the norm ‖. ‖ k are said to be D-compatible if and the metric d defined through the norm ‖. ‖ are D-compatible. A subset S of E is called Janhavi if the order relation and the metric d or the norm ‖. ‖ are D-compatible in it. In particular, if S = E, then E is called a Janhavi metric or Janhavi Banach space.

Definition

An upper semi-continuous and monotone nondecreasing function ψ: R+ → R+ is called a D-function provided ψ(0) = 0. A monotone operator T: E → E is called partially nonlinear D-contraction if there exists a D-function ψ such that

$$\|\mathcal{T}x - \mathcal{T}y\| \leq \psi(\|x - y\|) \quad (1)$$

for all comparable elements x, y ∈ E, where 0 < ψ(r) < r for r > 0. In particular, if ψ(r) = k r, k > 0, T is called a partial Lipschitz operator with a Lischitz constant k and moreover, if 0 < k < 1, T is called a partial linear contraction on E with a contraction constant k.

This study relies heavily on the Dhage iteration method, which is based on the following hybrid fixed point principle of Dhage in a partially ordered normed linear space.

Theorem 1

Let $E, \preceq, \|\cdot\|)$ be a regular partially ordered complete normed linear space such that every compact chain C of E is Janhavi. Let A,B : E → E be two nondecreasing operators such that

(a) A is a partially bounded and partially nonlinear D-contraction,

(b) B is partially continuous and partially compact,

(c) there exists an element α0 ∈ X such that

$$\alpha_0 \preceq \mathcal{A}\alpha_0 + \mathcal{B}\alpha_0 \text{or } \alpha_0 \succeq \mathcal{A}\alpha_0 + \mathcal{B}\alpha_0$$

Then the hybrid operator equation A x + B x = x has a solution x∗ and the sequence {x n} of successive iterations defined by x 0 = α 0, xn+1 = A x_n + B x_n, n = 0,1,...; converges monotonically to x^*.

Remark

The requirement that every compact chain of E is Janhavi is satisfied if every partially compact subset of E exhibits the compatibility property concerning the order relation and the norm k · k. This fundamental truth is employed to demonstrate the principal existence outcomes.

Remark

It is observed that the consistency of E in Theorem 1 can be substituted with a more robust continuity criterion for the operators A and B on E.

IV. MAIN EXISTENCE RESULTS

In this section, we prove an existence and approximation result for the HFDE on J = [a, b] under mixed partial Lipschitz and partial compactness type conditions on the nonlinearities involved in it. We place the HFDE in the function space C(J,R). We define a norm k · k an9d the order relation $\preceq$ in $C(J, \mathbb{R})$ by

$$\|x\| = \sup_{t \in J} |x(t)| \quad (2)$$

And

$x \preceq y \iff x(t) \leq y(t)$ For all t ∈ J. (3)

It is obvious that C(J,R) is a Banach space with respect to the supremum norm, in addition to being partially ordered in accordance with the partially ordered relation that is provided. The regular and lattice partially ordered Banach space C(J,R) assigns a lower bound and an upper bound to each and every pair of elements that are contained within E. When the compactness Arzela'-Ascoli theorem is taken into consideration, this useful lemma about Janhavi subsets of C(J,R) follows directly from it.

Lemma

Let $(C(J,\mathbb{R}), \preceq, \|\cdot\|)$ be a partially ordered Banach space with the norm ‖.‖ and the order relation defined by (2) and (3) respectively. Then every partially compact subset of C(J,R) is Janhavi.

Proof

The proof of the lemma is well-known and so we omit the details.

We introduce an order relation $\preceq_{\mathcal{C}}$ in $\mathcal{C}$ induced by the order relation defined in C(J,R). Thus, for any $x, y \in \mathcal{C}$, $x \preceq_{\mathcal{C}} y$ implies x(θ) ≤ y(θ) for all θ ∈ I0. Moreover, if x, y ∈ C(J,R) and $x \preceq y$, then $x_t \preceq_{\mathcal{C}} y_t$ for all t ∈ I.

Theorem 2

Suppose that hypotheses (H_1)-(H_2) and (H_4) hold. Then the HFDE has a solution x∗ defined on J and the sequence $\{x_n\}$ of successive approximations defined by

$$x_0 = u,$$
$$x_{n+1}(t) = \begin{cases} \phi(0) - f(0,\phi(0)) + f(t, x_n(t)) + \int_0^t g(s, x_s^n)\, ds, \\ \qquad\qquad if \;\; t \in I, \\ \phi(t), \quad if \;\; t \in I_0, \end{cases} \tag{4}$$

where $x_s^n(\theta) = x_n(s+\theta),\ \theta \in I_0$, converges monotonically to x∗.

Proof

Set E = C(J,R). Then, in view of Lemma every compact chain C in E possesses the compatibility property with respect to the norm k · k and the order relation ≤ so that every compact chain C is Janhavi in E.

Define two operators A and B on E by

$$\mathcal{A}x(t) = \begin{cases} -f(0,\phi(0)) + f(t, x(t)) \quad \text{if} \;\; t \in I, \\ 0, \quad \text{if} \;\; t \in I_0, \end{cases} \tag{5}$$

And

$$\mathcal{B}x(t) = \begin{cases} \phi(0) + \int_0^t g(s, x_s)\, ds, & \text{if } t \in I, \\ \phi(t), & \text{if } t \in I_0. \end{cases} \quad (6)$$

From the continuity of the functions f, g and the integral, it follows that A and B define the operators A,B : E → E. Applying Lemma the HFDE is equivalent to the operator equation

A x (t) + B x (t) = x(t), t ∈ J. (7)

V. HYBRID INTEGRODIFFERENTIAL EQUATIONS

In this section, we consider the following nonlinear first order hybrid functional integrodifferential equation (in short HFIDE) of neutral type

$$\left.\begin{aligned} \frac{d}{dt}\left[x(t) - f\left(t, x(t), x_t\right)\right] &= g\left(t, x_t, \int_0^t k(s, x_s)\, ds\right),\ t \in I, \\ x_0 &= \phi, \end{aligned}\right\} \quad (8)$$

where φ ∈ C and f : I × R × C → R , k : I × C → R and g : I × C × R → R are continuous functions.

Definition

A function x ∈ C(J,R) is said to be a solution of the HFIDE if

(i) x_0 ∈ C,

(ii) X t ∈ C for each t ∈ I, and

(iii) the function $t \mapsto \left[x(t) - f\left(t, x(t), x_t\right)\right]$ is continuously differentiable on I and satisfies the equation in (8).

The neutral HFIDE is a new linear perturbation of the second type of functional differential equation. It incorporates a number of functional differential equations that were investigated earlier in Dhage as special cases. In this chapter, we demonstrate the existence and approximation theorem for the hybrid functional differential equations of neutral type (8) by employing a novel iteration method. This method does not necessitate the existence of both upper and lower solutions, nor does it require the monotonic inequality that is associated with them. Furthermore, we obtain the algorithm for the solutions under certain natural conditions.

VI. CONCLUSION

We have seen that the Dhage iteration method is a powerful and intriguing tool for discussing the existence results of specific nonlinear hybrid quadratic fractional differential equations in the previous discussion. Unfortunately, there are a few drawbacks to the new method. One of them is that, in contrast to Picard's method, it does not provide the rate of convergence of the sequence of successive approximations. The numerical mild solution of the nonlinear fractional differential equation under consideration has been proven, however. To wrap up this paper, we'd like to mention that the quadratic fractional differential equation we're looking at is extremely simple, and that we used the Dhage iteration method to find algorithms for mild solutions under weaker partially Lipschitz and compactness conditions. But other complicated quadratic fractional differential equations could also be studied in a similar way, with the right adjustments. There will be other reports detailing some of the outcomes related to this.

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